Geometry 4-7 Study Guide: Triangle Congruence: CPCTC (pp 268-273)

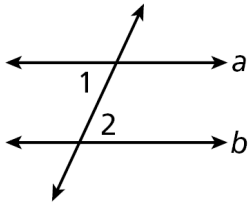
Attendance Problems.

1. If $\triangle ABC \cong \triangle DEF$, then $\measuredangle A \cong$ _____ & $\overline{BC} \cong$ _____.

2. What is the distance between (3, 4) & (1, 5)?

3. If $\measuredangle 1 \cong \measuredangle 2$, why is a || b?

4. List the methods used to prove two triangles congruent.



I can use CPCTC to prove parts of triangles are congruent.

Vocabulary: CPCTC

Common Core

CC.9-12.G.SRT.5 Use congruence and similarity criteria for triangles to solve problems and prove relationships in geometric figures.

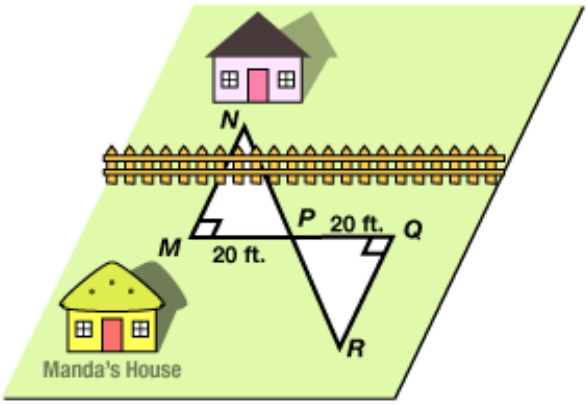
CC.9-12.G.MG.3 Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).*

CPCTC is an abbreviation for the phrase "Corresponding Parts of Congruent Triangles are Congruent." It can be used as a justification in a proof after you have proven two triangles congruent.

Remember!

SSS, SAS, ASA, AAS, and HL use corresponding parts to prove triangles congruent. CPCTC uses congruent triangles to prove corresponding parts congruent.

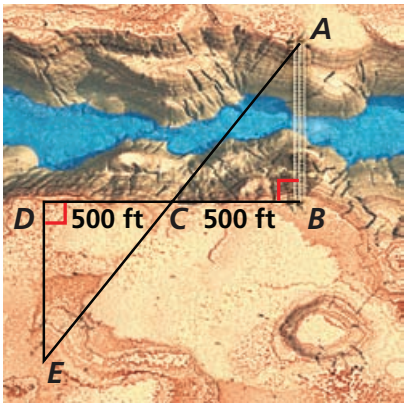
Video Example 1. Manda wants to know the distance between her front door and her neighbor's. She located points P, Q, and R. How can she find MN?



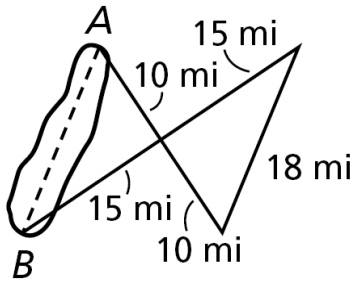
1 Engineering Application

To design a bridge across a canyon, you need to find the distance from *A* to *B*. Locate points *C*, *D*, and *E* as shown in the figure. If *DE* = 600 ft, what is *AB*?

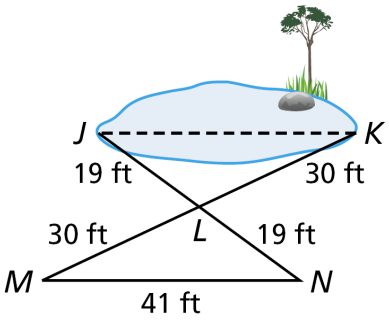
$\angle D \cong \angle B$, because they are both right angles.
$\overline{DC} \cong \overline{CB}$, because $DC = CB = 500$ ft.
$\angle DCE \cong \angle BCA$, because vertical angles are congruent. Therefore $\triangle DCE \cong \triangle BCA$ by ASA or LA. By CPCTC, $\overline{ED} \cong \overline{AB}$, so $AB = ED = 600$ ft.



Example 1. A and B are on the edges of a ravine. What is AB?



5. Guided Practice. A landscape architect sets up the triangles shown in the figure to find the distance *JK* across a pond. What is *JK*?

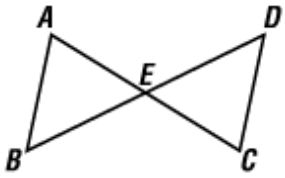


Video Example 2.

Given: $\overline{AE} \cong \overline{EC}$

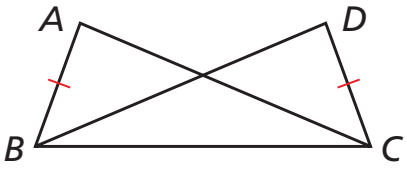
$\overline{BE} \cong \overline{ED}$

Prove: $\overline{AB} \cong \overline{DC}$

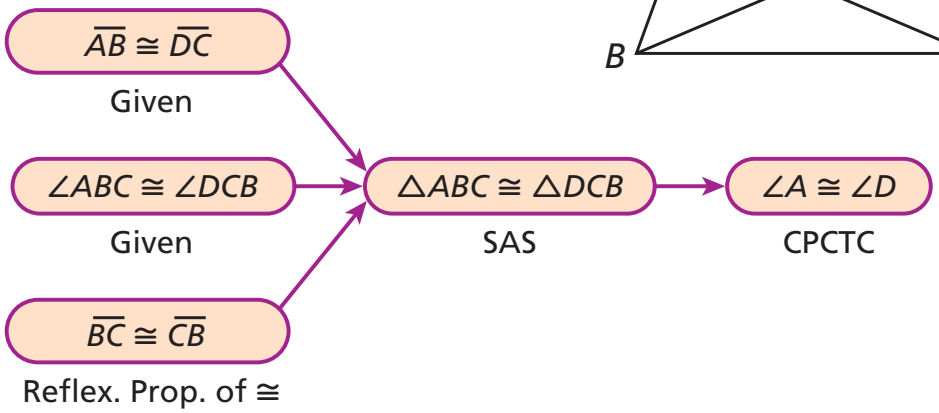


2 Proving Corresponding Parts Congruent

Given: $\overline{AB} \cong \overline{DC}$, $\angle ABC \cong \angle DCB$
Prove: $\angle A \cong \angle D$

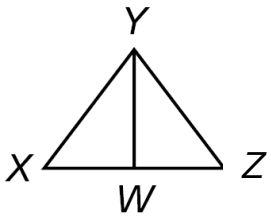


Proof:



Example 2.

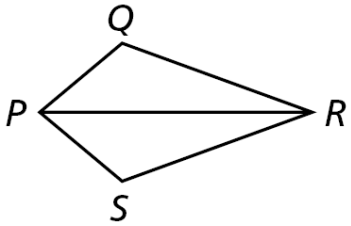
Given: $\overline{YW}$ *bisects* $\overline{XZ}$
$\overline{XY} \cong \overline{YZ}$
Prove: $\measuredangle XYW \cong \measuredangle ZYW$



6. Guided Practice.

Given: $\overline{PR}$ bisects $\measuredangle QPS$ & $\measuredangle QRS$.

Prove: $\overline{PQ} \cong \overline{PS}$



Helpful Hint

Work backward when planning a proof. To show that $\overline{ED}$ || $\overline{GF}$, look for a pair of angles that are congruent.

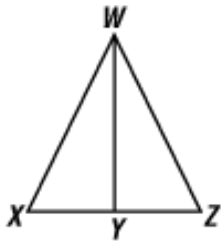
Then look for triangles that contain these angles.

I think that all right-thinking people in this country are sick and tired of being told that ordinary decent people are fed up in this country with being sick and tired. I'm certainly not. But I'm sick and tired of being told that I am. - *Monty Python*

Video Example 3.

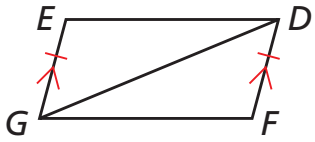
Given: Y is the midpoint $\overline{XZ}$.
$\measuredangle XYW$ is a right angle.

Prove: $\measuredangle XWY \cong \measuredangle ZWY$



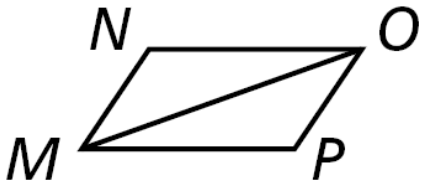
3 Using CPCTC in a Proof

Given: $\overline{EG} \parallel \overline{DF}$, $\overline{EG} \cong \overline{DF}$
Prove: $\overline{ED} \parallel \overline{GF}$
Proof:



Statements	Reasons
1. $\overline{EG} \cong \overline{DF}$	1. Given
2. $\overline{EG} \parallel \overline{DF}$	2. Given
3. $\angle EGD \cong \angle FDG$	3. Alt. Int. $\measuredangle$s Thm.
4. $\overline{GD} \cong \overline{DG}$	4. Reflex. Prop. of $\cong$
5. $\triangle EGD \cong \triangle FDG$	5. SAS *Steps 1, 3, and 4*
6. $\angle EDG \cong \angle FGD$	6. CPCTC
7. $\overline{ED} \parallel \overline{GF}$	7. Converse of Alt. Int. $\measuredangle$s Thm.

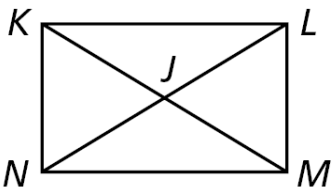
Example 3.
Given: $\overline{NO} \parallel \overline{MP}$
$\measuredangle N \cong \measuredangle P$
Prove: $\overline{MN} \cong \overline{OP}$



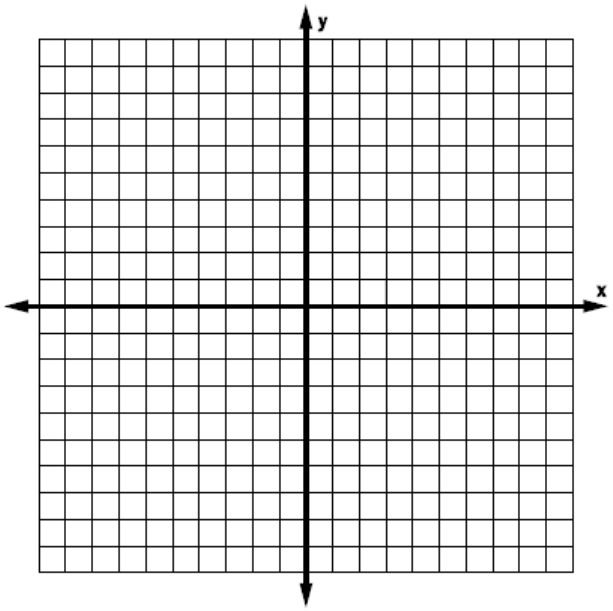
7. <u>Guided Practice</u>.

Given: J is the midpoint of $\overline{KM}$ & $\overline{NL}$.

Prove: $\overline{KL} \parallel \overline{MN}$



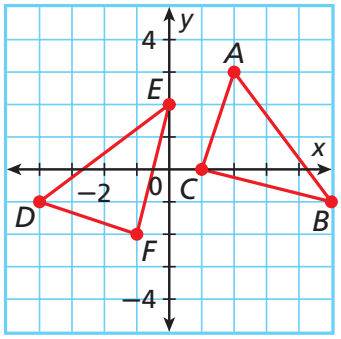
<u>Video Example 4</u>.
Given: A(-3, -6), B(-3, 6), C(-7, 4), D(3, 8), E(3, -4), F(7, -2)
Prove: $\measuredangle BCA \cong \measuredangle DFE$



4 Using CPCTC in the Coordinate Plane

Given: $A(2, 3), B(5, -1), C(1, 0),$
$D(-4, -1), E(0, 2), F(-1, -2)$

Prove: $\angle ABC \cong \angle DEF$



Step 1 Plot the points on a coordinate plane.

Step 2 Use the Distance Formula to find the lengths of the sides of each triangle.

$$D = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$AB = \sqrt{(5 - 2)^2 + (-1 - 3)^2} = \sqrt{9 + 16} = \sqrt{25} = 5$$

$$BC = \sqrt{(1 - 5)^2 + (0 - (-1))^2} = \sqrt{16 + 1} = \sqrt{17}$$

$$AC = \sqrt{(1 - 2)^2 + (0 - 3)^2} = \sqrt{1 + 9} = \sqrt{10}$$

$$DE = \sqrt{(0 - (-4))^2 + (2 - (-1))^2} = \sqrt{16 + 9} = \sqrt{25} = 5$$

$$EF = \sqrt{(-1 - 0)^2 + (-2 - 2)^2} = \sqrt{1 + 16} = \sqrt{17}$$

$$DF = \sqrt{(-1 - (-4))^2 + (-2 - (-1))^2} = \sqrt{9 + 1} = \sqrt{10}$$

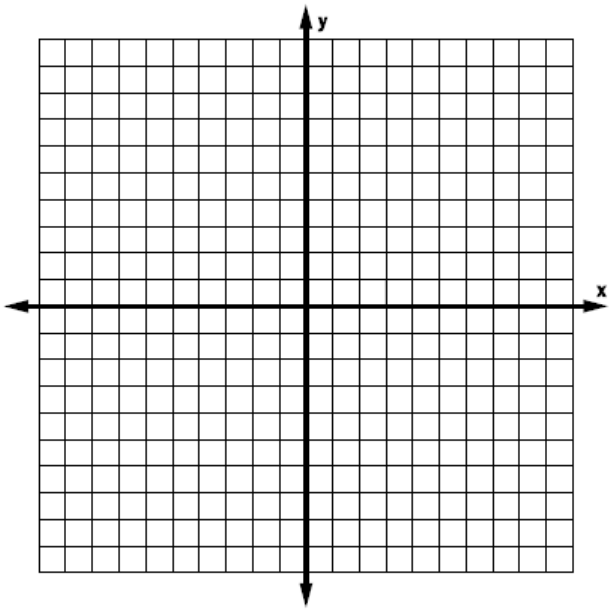
So $\overline{AB} \cong \overline{DE}$, $\overline{BC} \cong \overline{EF}$, and $\overline{AC} \cong \overline{DF}$. Therefore $\triangle ABC \cong \triangle DEF$ by SSS, and $\angle ABC \cong \angle DEF$ by CPCTC.

"Of students surveyed, 64% prefer English and 32% prefer math. The fact that these numbers do not add up to 100 may help explain why."

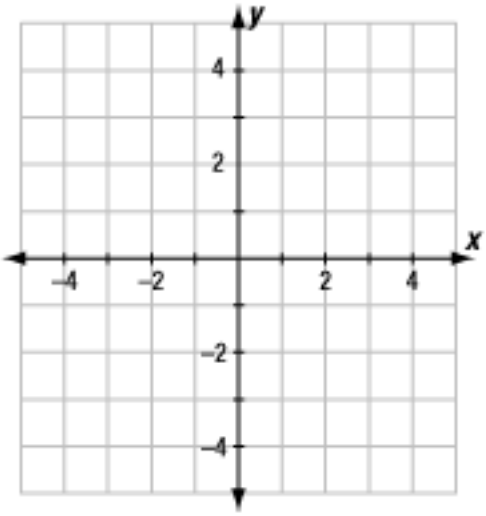
Example 4.

Given: $D(-5, -5)$, $E(-3, -1)$, $F(-2, -3)$, $G(-2, 1)$, $H(0, 5)$, and $I(1, 3)$

Prove: $\measuredangle DEF \cong \measuredangle GHI$



8. <u>Guided Practice</u>.
Given: $J(-1, -2)$, $K(2, -1)$, $L(-2, 0)$, $R(2, 3)$, $S(5, 2)$, $T(1, 1)$
Prove: $\measuredangle JKL \cong \measuredangle RST$



4-7 Triangle Congruence: CPCTC

(*pp 271)* 7-13, 14-18 even, 22-24.
(pp 277) 4-8.